Energy Harvesting Wireless Communication Networks

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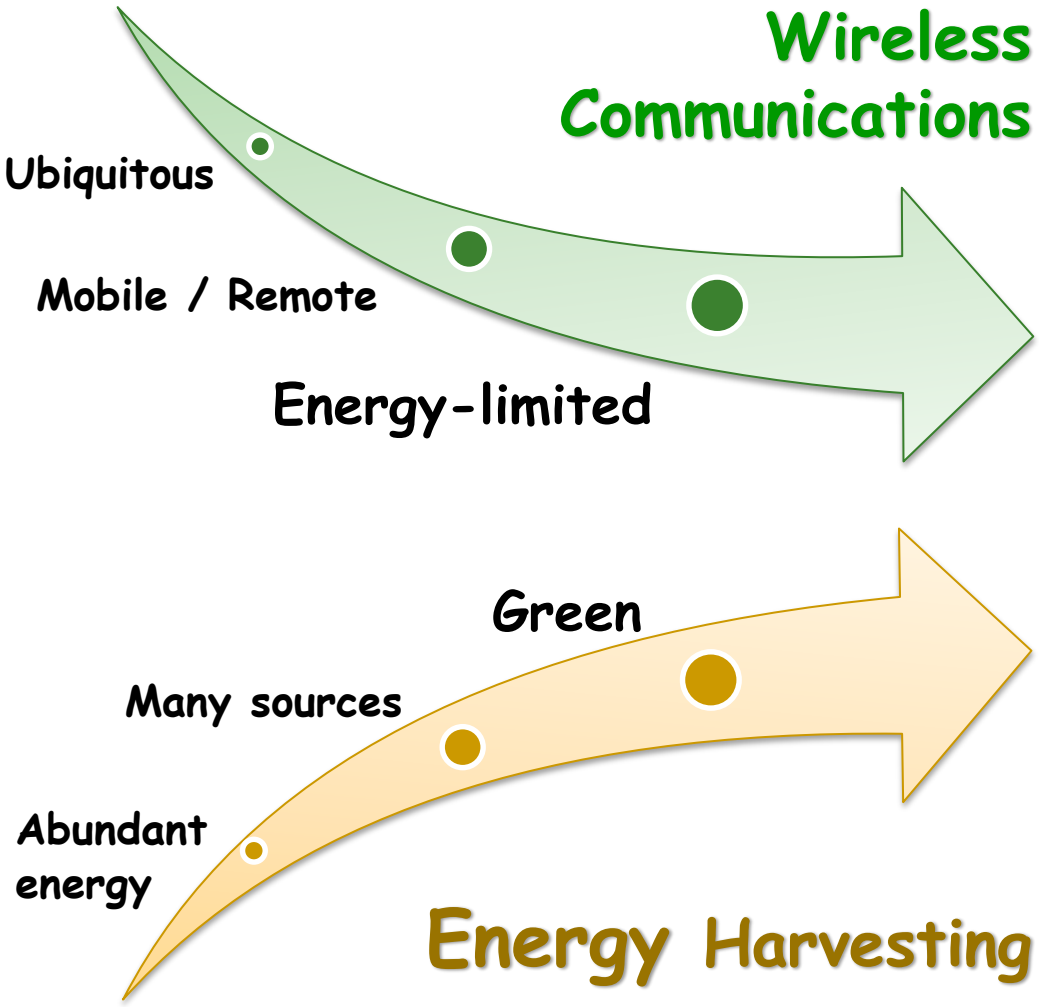
Introduction



Energy Harvesting Wireless Networks

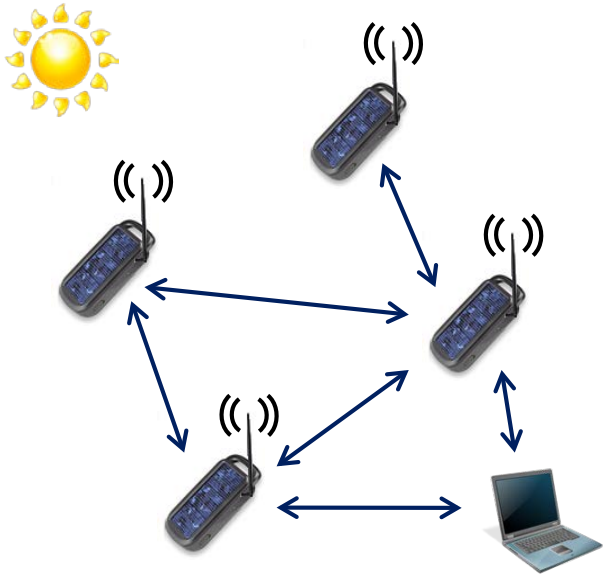
Energy Harvesting Networks

- **Wireless networking with rechargeable (energy harvesting) nodes:**
  - Green, self-sufficient nodes,
  - Extended network lifetime,
  - Smaller nodes with smaller batteries.

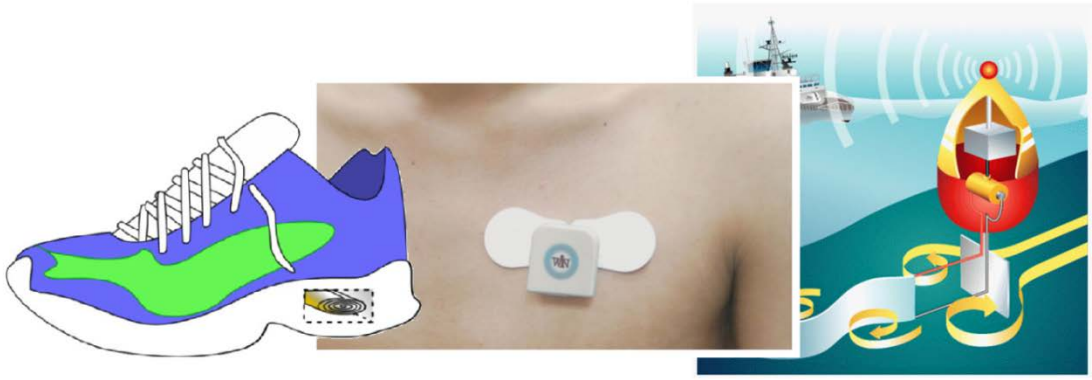
A relatively new field with increasing interest.

Some Applications

Wireless sensor networks

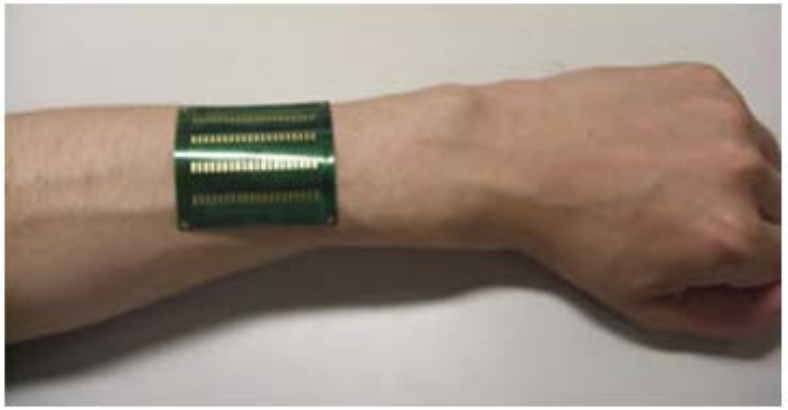


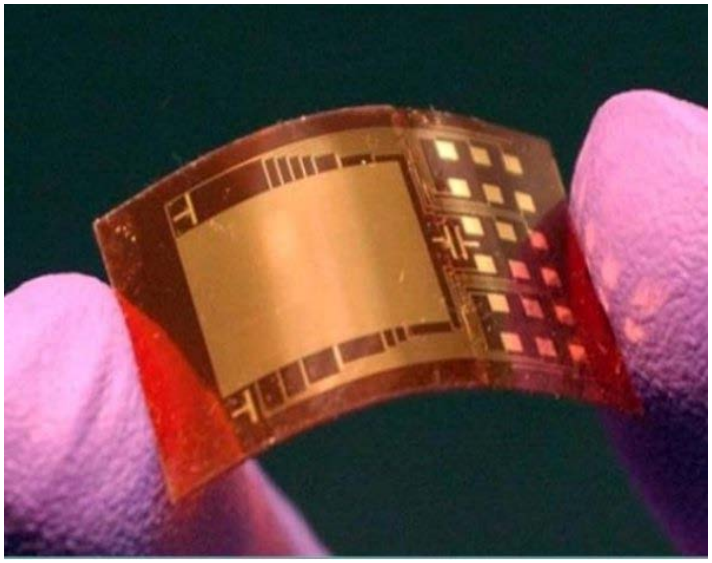
Green communications



Harvesting Energy

- Fujitsu's hybrid device utilizing **heat or light.**





- Nanogenerators built at Georgia Tech, utilizing **strain**

Image Credits: (top) http://www.fujitsu.com/global/news/pr/archives/month/2010/20101209-01.html
(bottom) http://www.zeitnews.org/nanotechnology/squeeze-power-first-practical-nanogenerator-developed.html

New Network Design Challenge

- A set of energy feasibility constraints based on energy harvests govern the communication resources.
- **Main design question:**

  When and at what rate/power should a rechargeable (energy harvesting) node transmit?
- **Optimality? Throughput; Delivery Delay**
- **Outcome: Optimal Transmission Schedules**

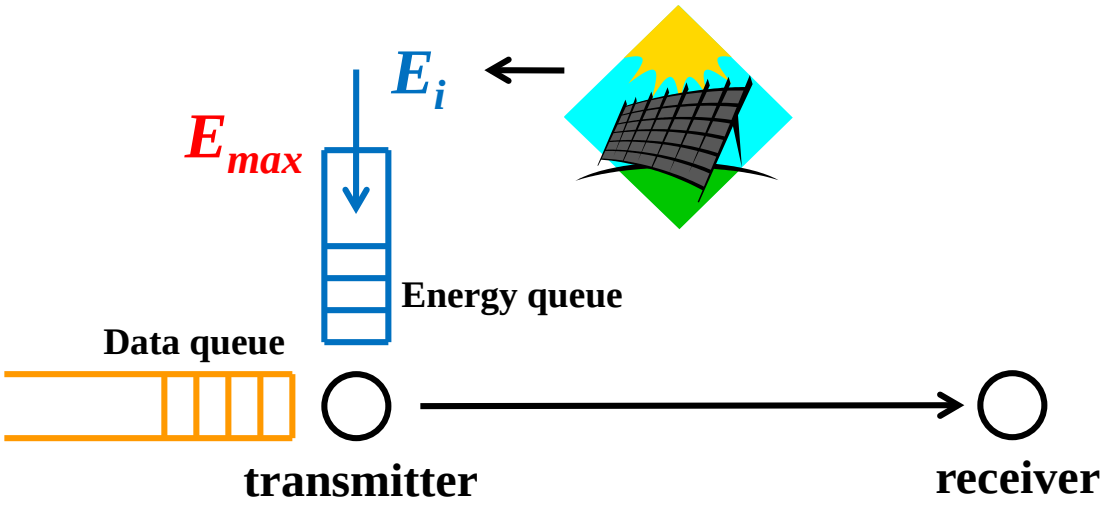
Throughput Maximization

[Tutuncuoglu-Y.'12]

- One Energy harvesting transmitter.
- Find optimal power allocation/transmission policy that departs maximum number of bits in a given duration **T**.
- Energy available intermittently.
- Up to a certain amount of energy can be stored by the transmitter → **BATTERY CAPACITY.**

System Model

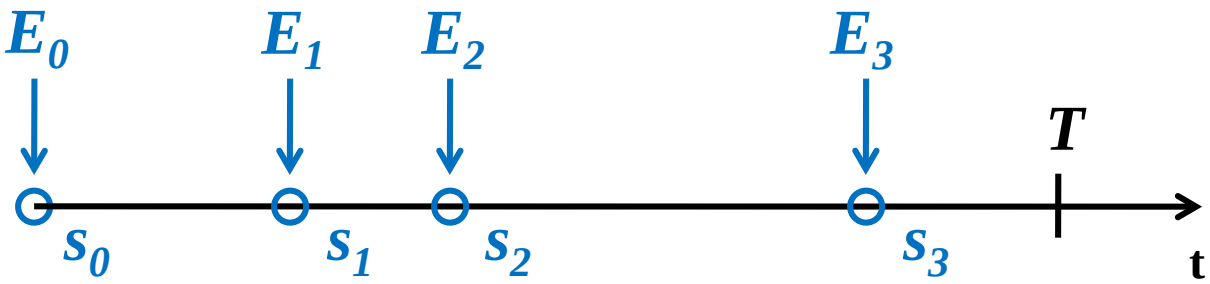
- Energy harvesting transmitter:



- Energy **arrives intermittently** from harvester
- Transmitter has **backlogged data** to send within a deadline T.
- Stored in a **finite battery** of capacity E_{max}

System Model

- Energy arrivals of energy E_i at times s_i



- Arrivals known **non-causally** by transmitter,
- Design parameter: **power** $\rightarrow$ **rate** $r(p)$.

Notations and Assumptions

- Power allocation function: $p(t)$
- Energy consumed: $\int_0^T p(t)dt$
- Transmission with power p yields a rate of $r(p)$
- Short-term throughput: $\int_0^T r(p(t))dt$

Power-Rate Function

- Transmission with power p yields a rate of $r(p)$

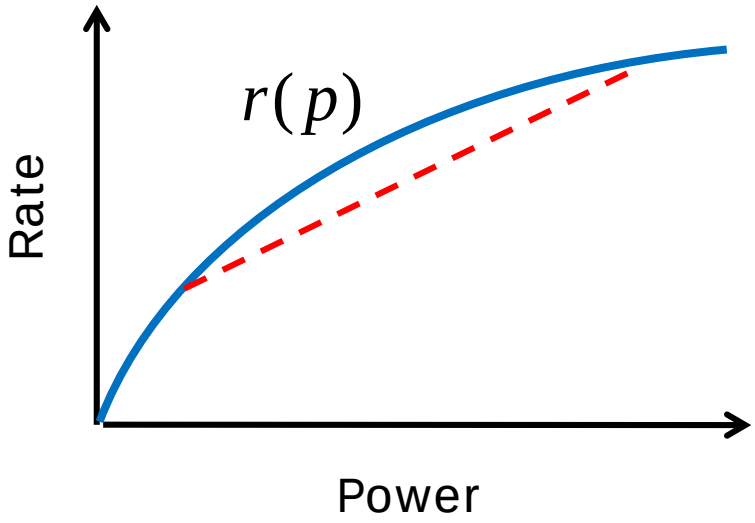
- **Assumptions on r(p):**

  i. $r(0)=0,\ r(p) \to \infty\ as\ p \to \infty$

  ii. increases monotonically in p

  iii. **strictly concave**

  iv. $r(p)$ continuously differentiable



Example: AWGN Channel, $r(p) = \frac{1}{2}\log\left(1+\frac{p}{N}\right)$

Energy Constraints

(Energy arrivals of E_i at times s_i)

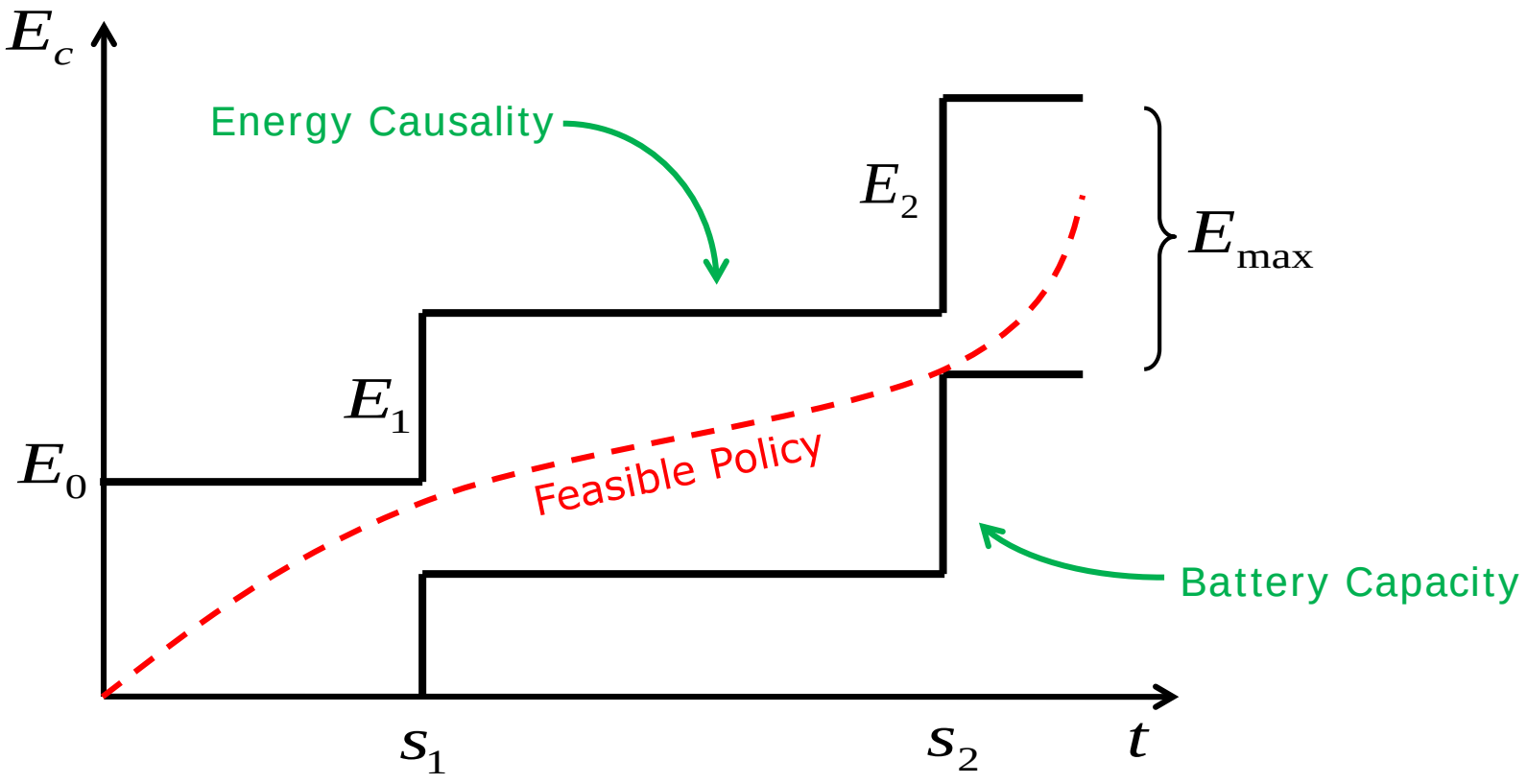
- **Energy Causality:** $\sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \geq 0 \qquad s_{n-1} \leq t' \leq s_n$

- **Battery Capacity:** $\sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \leq E_{\max} \qquad s_{n-1} \leq t' \leq s_n$

- **Set of energy-feasible power allocations**

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \leq \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \leq E_{\max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}$$

Energy "Tunnel"



Optimization Problem

- Maximize total number of transmitted bits by deadline T

$$\max_{p(t)} \int_0^T r(p(t))dt, \qquad s.t. \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{i=0}^{n-1} E_i - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

- Convex constraint set, concave maximization problem

Necessary conditions for optimality of a transmission policy

- **Property 1:** Transmission power remains constant between energy arrivals.

Transmission power can change only at S_i

Necessary conditions for optimality

- **Property 2:** Battery never overflows.

Proof:

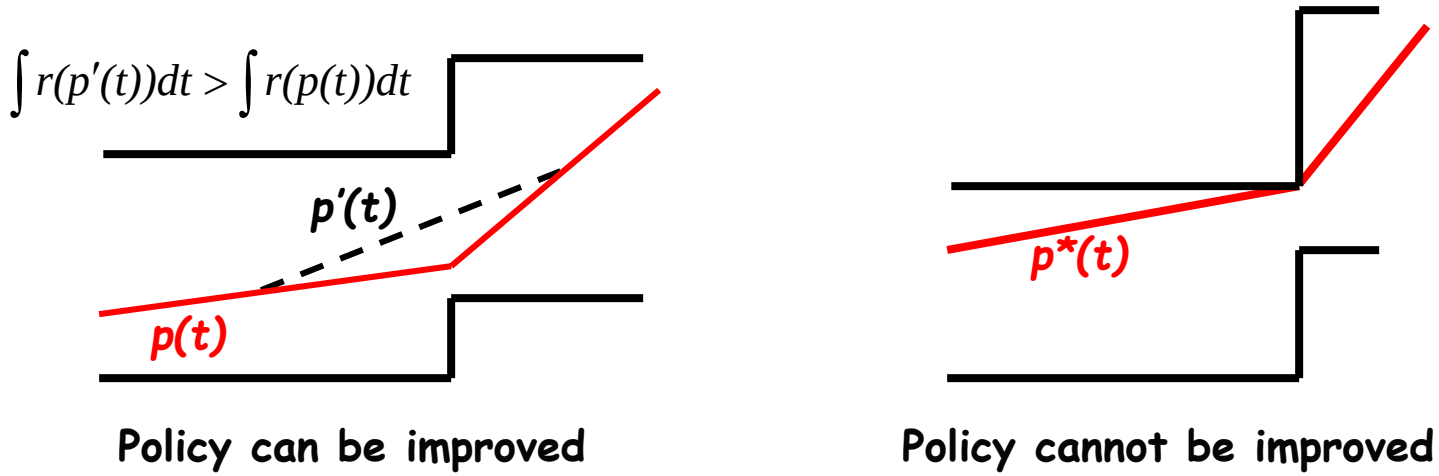
Assume an energy of Δ overflows at time τ

Define $p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [\tau - \delta, \tau] \\ p(t) & else \end{cases}$

Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))dt$ since $r(p)$ is increasing in p

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.

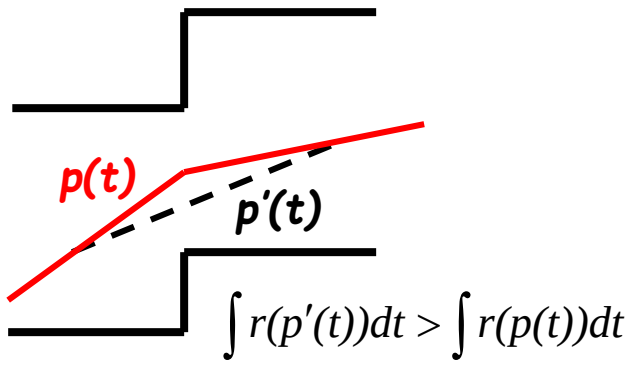


Policy can be improved

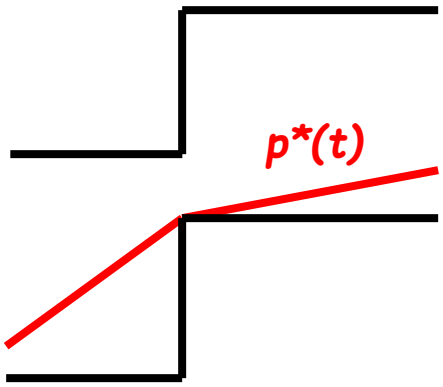
Policy cannot be improved

Necessary conditions for optimality of a transmission policy

- **Property 3:** Power level increases at an energy arrival instant only if battery is depleted. Conversely, power level decreases at an energy arrival instant only if battery is full.



Policy can be improved



Policy cannot be improved

Necessary conditions for optimality of a transmission policy

- **Property 4:** Battery is depleted at the end of transmission.

  Proof: Assume an energy of Δ remains after p(t)

  Define $p'(t) = \begin{cases} p(t) + \dfrac{\Delta}{\delta} & [T-\delta, T] \\ p(t) & else \end{cases}$

  Then $\int_0^T r(p'(t))dt > \int_0^T r(p(t))dt$ since $r(p)$ is increasing

Necessary Conditions for Optimality

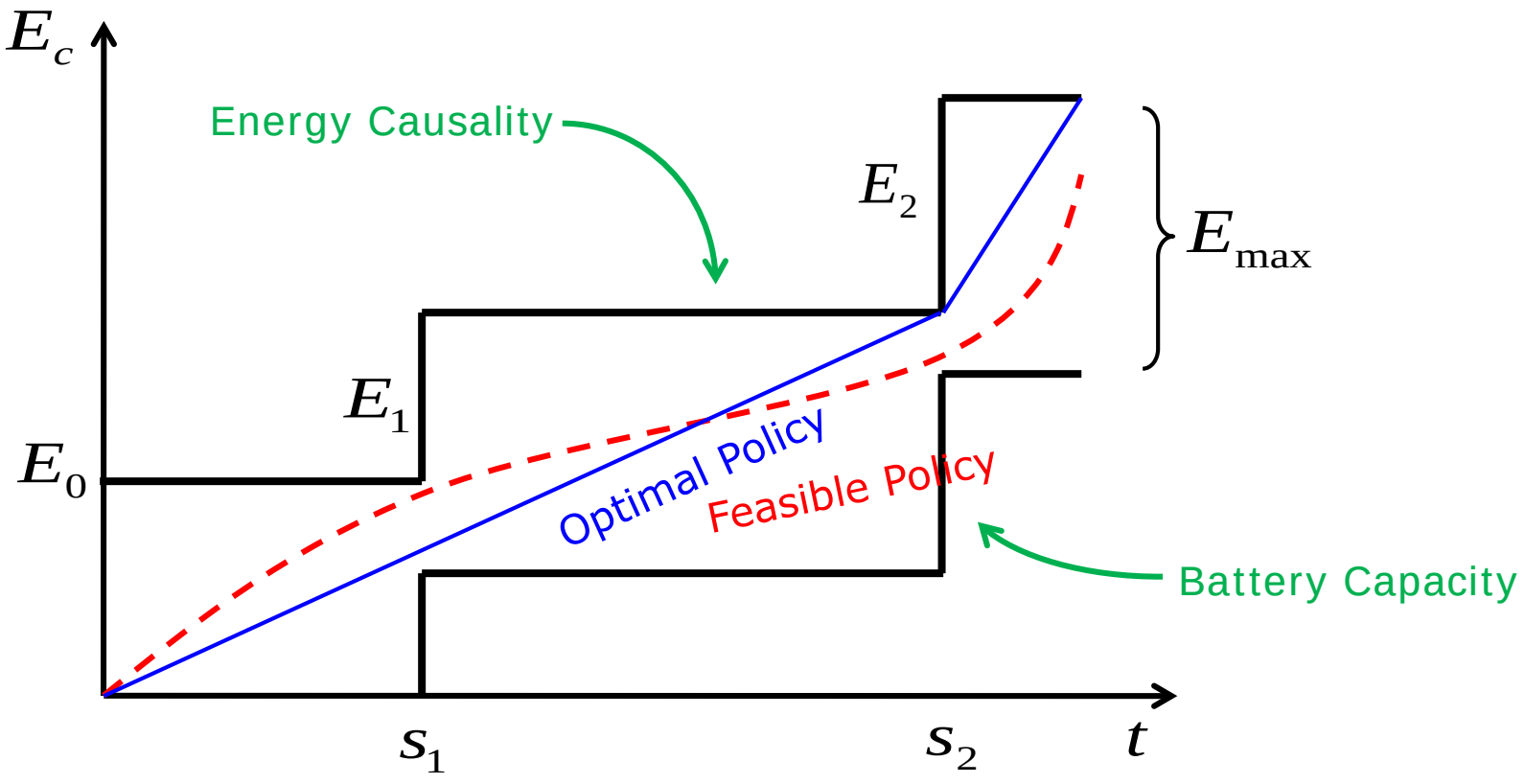
Implications of Properties 1-4:

- **Structure of optimal policy:** (Property 1)

$$p(t) = \begin{cases} p_n & i_{n-1} < t < i_n \\ 0 & t > T \end{cases}, \qquad i_n \in \{s_n\}, \quad p_n \text{ constant}$$

- For power to increase or decrease, policy must meet the upper or lower boundary of the tunnel respectively (Property 3)
- At termination step, battery is depleted (Property 4).
- An algorithmic solution can be found recursively, see [Tutuncuoglu-Y.12]

Energy "Tunnel"



Shortest Path Interpretation

- **Optimal policy is identical for any concave power-rate function!**
- Let $r(p) = -\sqrt{p^2+1}$, then the problem solved becomes:

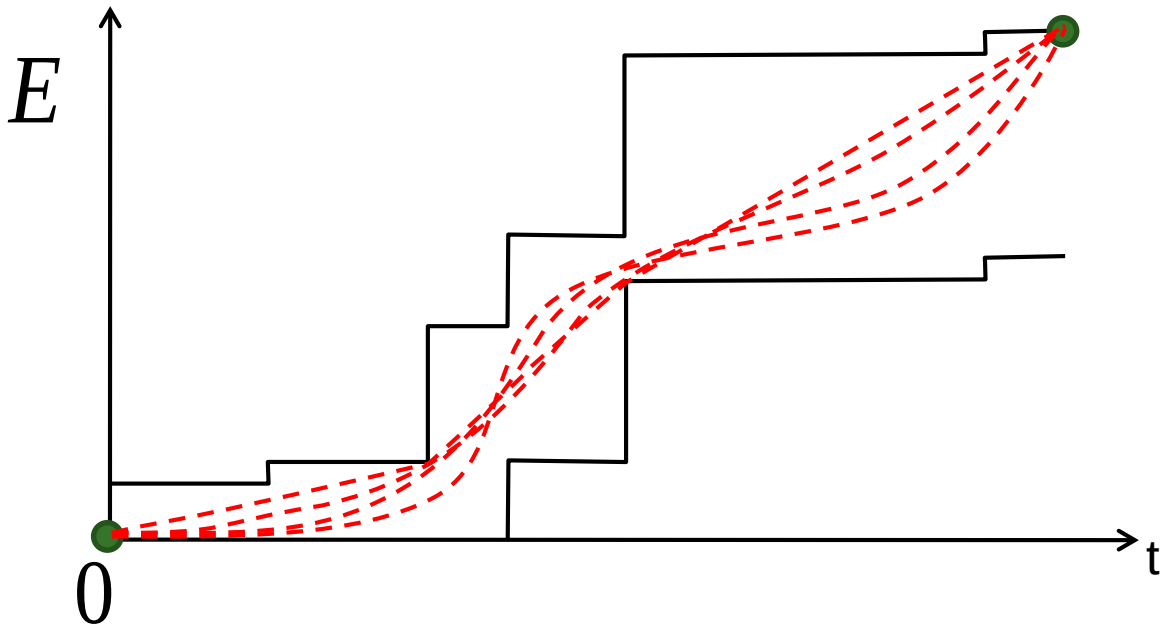
$$\max_{p(t)} \int_0^T -\sqrt{p^2(t)+1}\, dt \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

$$= \min_{p(t)} \underbrace{\int_0^T \sqrt{p^2(t)+1}\, dt}_{\textbf{length of policy path in energy tunnel}} \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

$\Rightarrow$ The **throughput maximizing policy** yields the **shortest path** through the energy tunnel for any concave power-rate function.

Shortest Path Interpretation

- **Property 1**: Constant power is better than any other alternative
- **Shortest path** between two points is a line (constant slope)



Alternative Solution (Using Property 1)

- Transmission power is constant within each epoch:

$$p(t) = \{p_i, t \in epoch\ i,\ \ i = 1,\ldots,N\}$$

(N: Number of arrivals within [0,T])

$$\max_{p_i} \sum_{i=1}^{N} L_i.r(p_i)$$

(L_i: length of epoch i)

$$s.t. \quad 0 \le \sum_{i=1}^{n} E_i - L_i p_i \le E_{\max} \quad n = 1,\ldots,N$$

- KKT conditions → optimum power policy.

Solution

- Complementary Slackness Conditions:

$$\lambda_n \left(\sum_{i=1}^{n} L_i p_i - E_i \right) = 0 \qquad \forall n$$

$$\mu_n \left(\sum_{i=1}^{n} E_i - L_i p_i - E_{\max} \right) = 0 \qquad \forall n$$

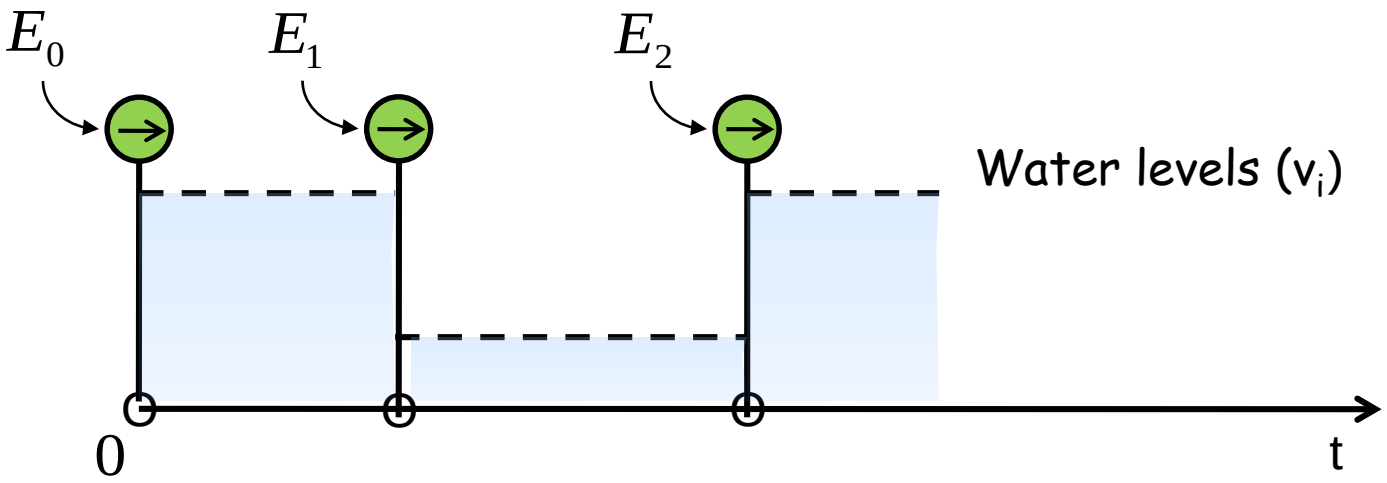
λ_n is positive only when battery is empty $\left(\sum_{i=1}^{n} L_i p_i - E_i \right) = 0$

μ_n is positive only when battery is full $\left(\sum_{i=1}^{n} E_i - L_i p_i - E_{\max} \right) = 0$

$$p_n^* = \frac{1}{\sum_{i=n}^{N} (\lambda_i - \mu_i)} - 1$$

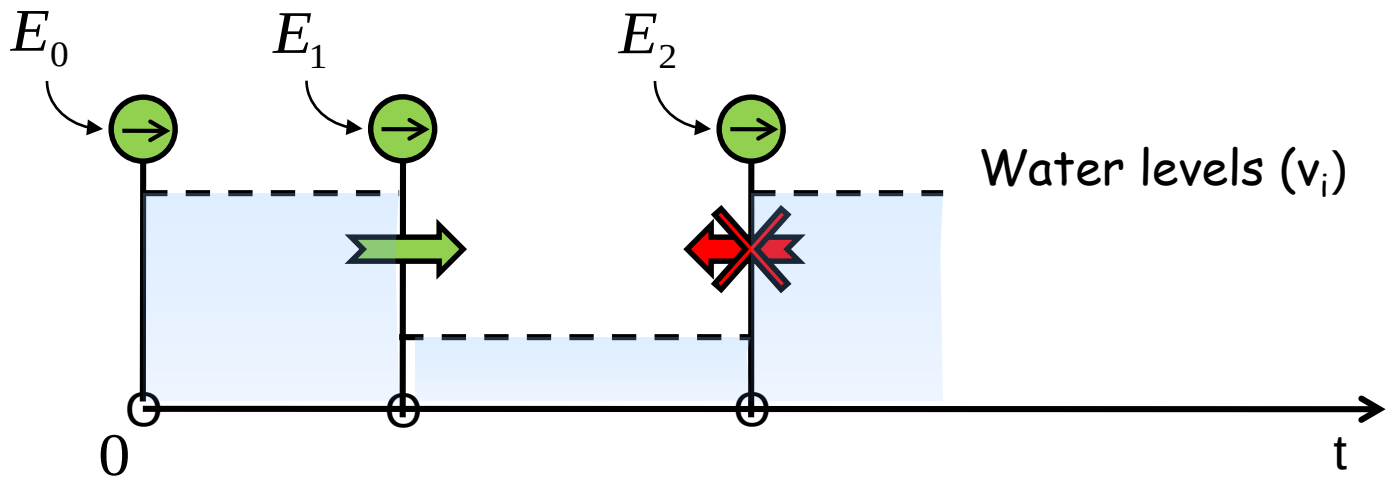
Directional Water-Filling

- [Ozel, Tutuncuoglu, Ulukus, Y., 2011]
- Harvested energies filled into epochs individually



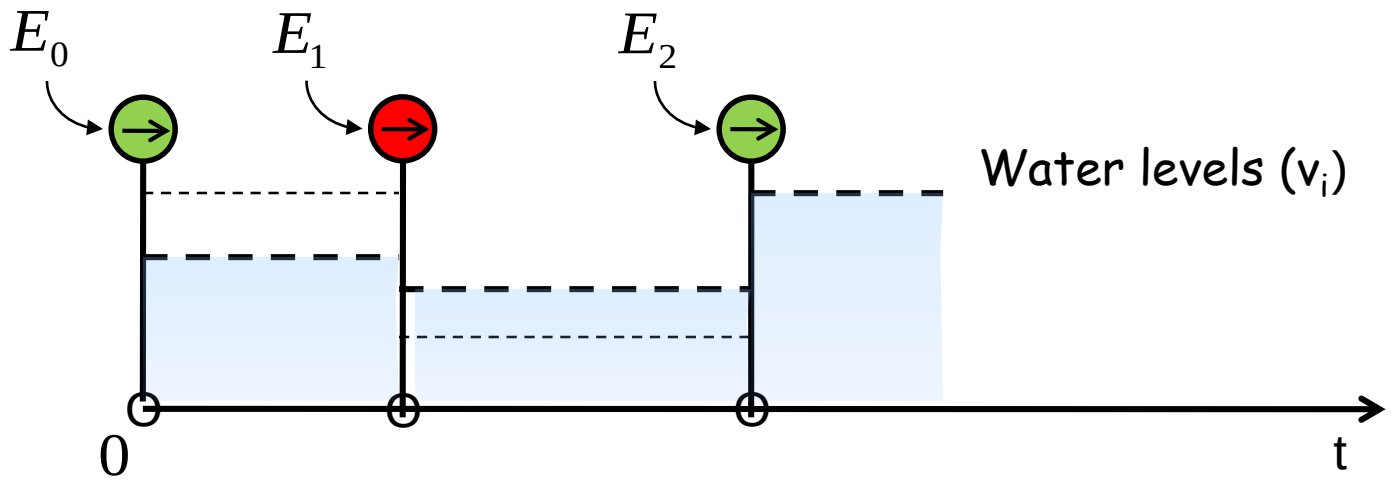
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality:** water-flow only forward in time ⟹



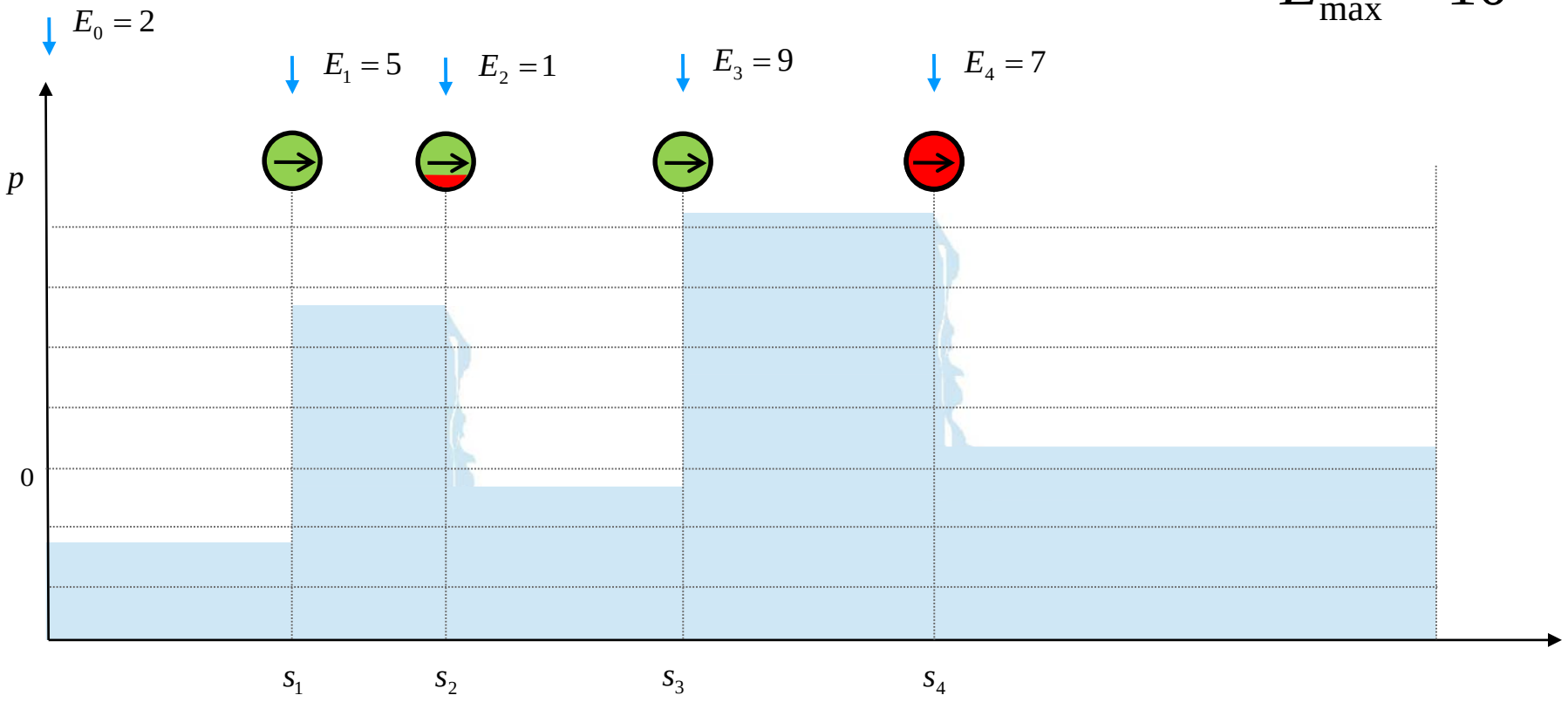
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality:** water-flow only forward in time
  - **Battery Capacity:** water-flow limited to E_{max} by taps

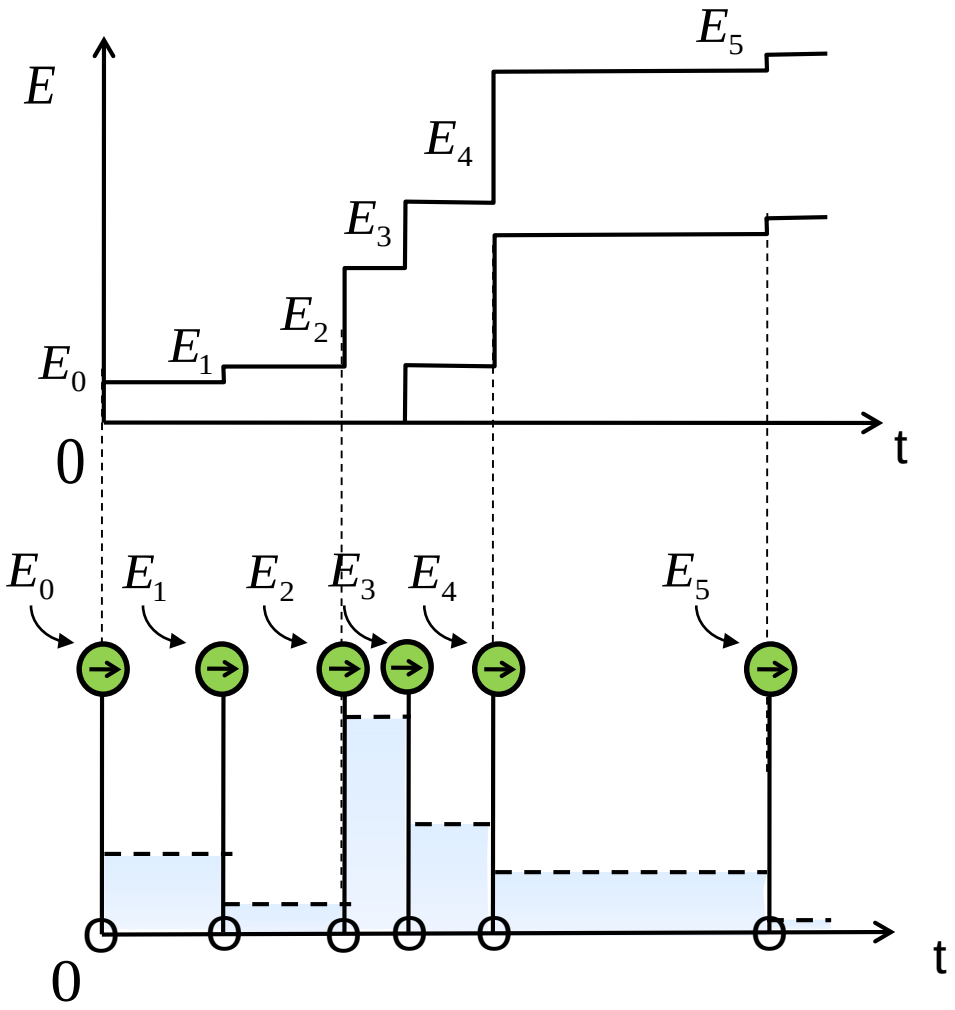


Example

$E_{\max} = 10$

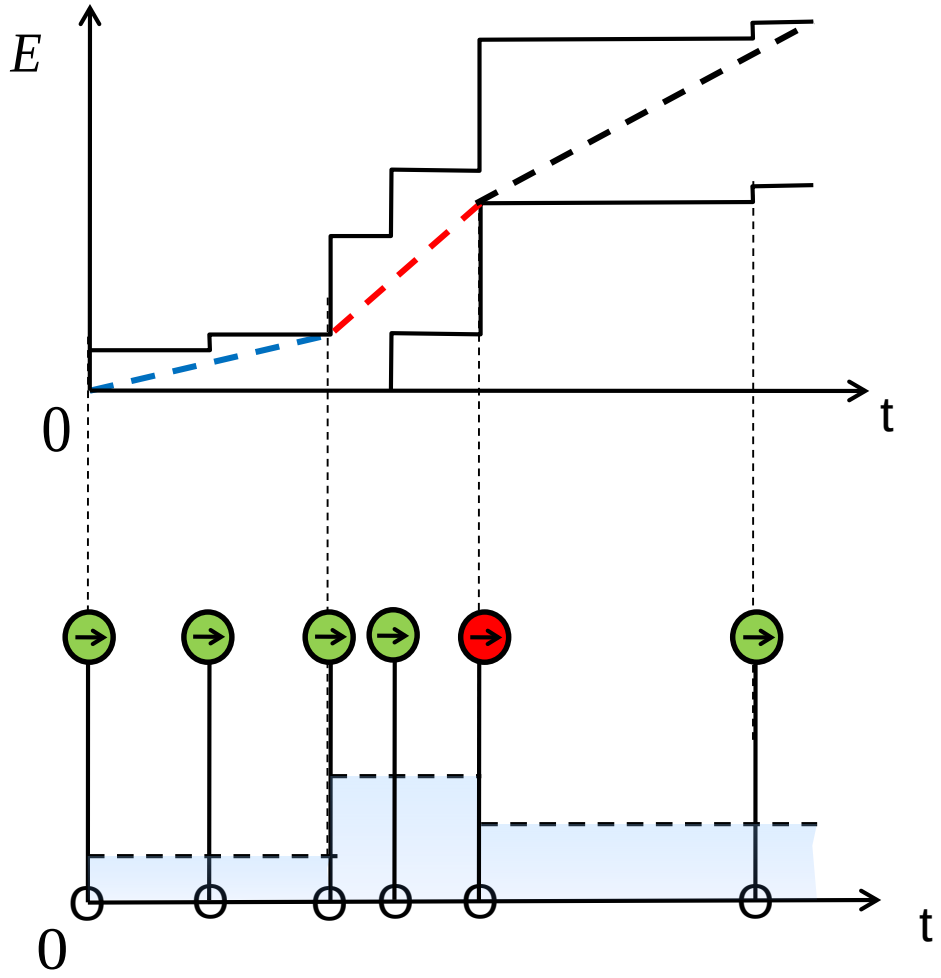


Directional Water-Filling



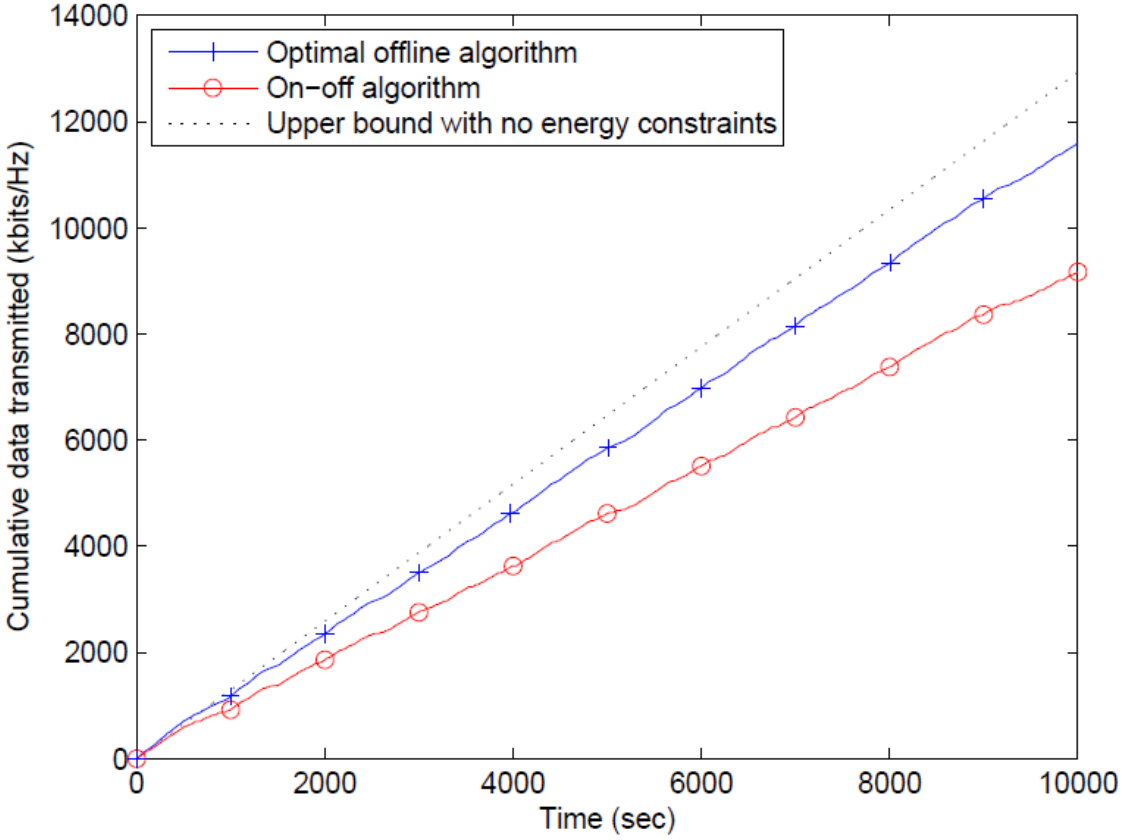
- Energy tunnel and directional water-filling approaches yield the same policy

Directional Water-Filling



- Energy tunnel and directional water-filling approaches yield the same policy

Simulation Results



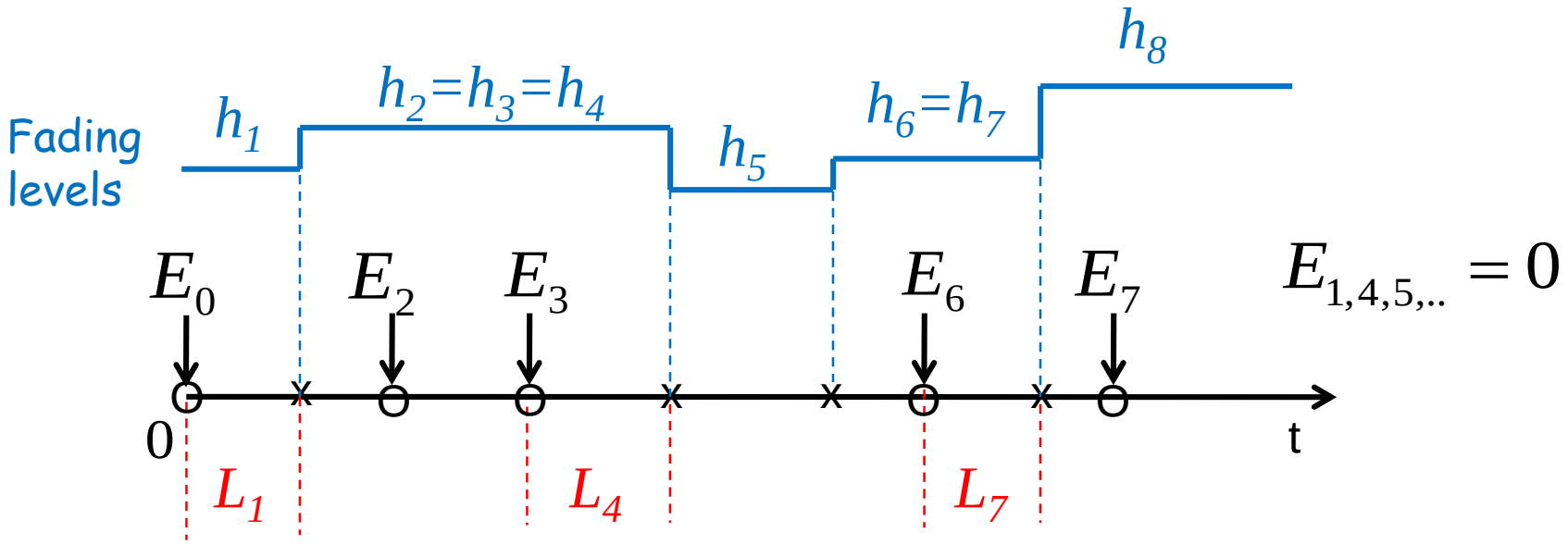
- Improvement of optimal algorithm over an *on-off transmitter* in a simulation with truncated Gaussian arrivals.

Extension to Fading Channels

- [Ozel-Tutuncuoglu-Ulukus-Y.'11]

- Find the short-term throughput maximizing and transmission completion time minimizing power allocations in a fading channel with known channel states.
- Finite battery capacity

System Model



- AWGN Channel with fading h: $r(p,h)=\frac{1}{2}\log(1+hp)$
- Each "epoch" defined as the interval between two "events".

TM Problem with Fading

- Transmission power constant within each epoch:

$$p(t) = \{p_i, t \in epoch\ i,\ \ i = 1,\ldots,M\}$$

- Maximize total number of transmitted bits by deadline T

$$\max_{p_i} \sum_{i=1}^{M} \frac{L_i}{2} \log(1 + h_i p_i)$$

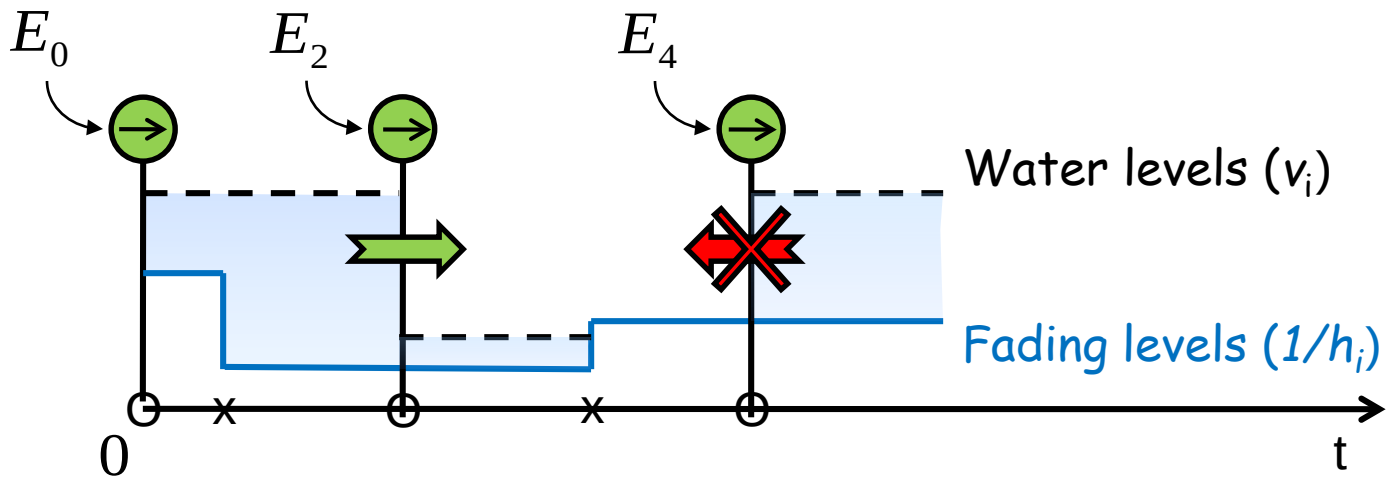
$$s.t. \quad 0 \le \sum_{i=1}^{n} E_i - L_i p_i \le E_{\max} \quad n = 1,\ldots,M$$

- Solution once again is directional waterfilling.

$$p_n^* = \left[\frac{1}{\sum_{i=n}^{M} (\lambda_i - \mu_i)} - \frac{1}{h_n} \right]^+$$

Directional Water-Filling for fading channels

- Same directional water filling model with added fading levels.
  - Directional water flow **(Energy causality)**
  - Limited water flow **(Battery capacity)**

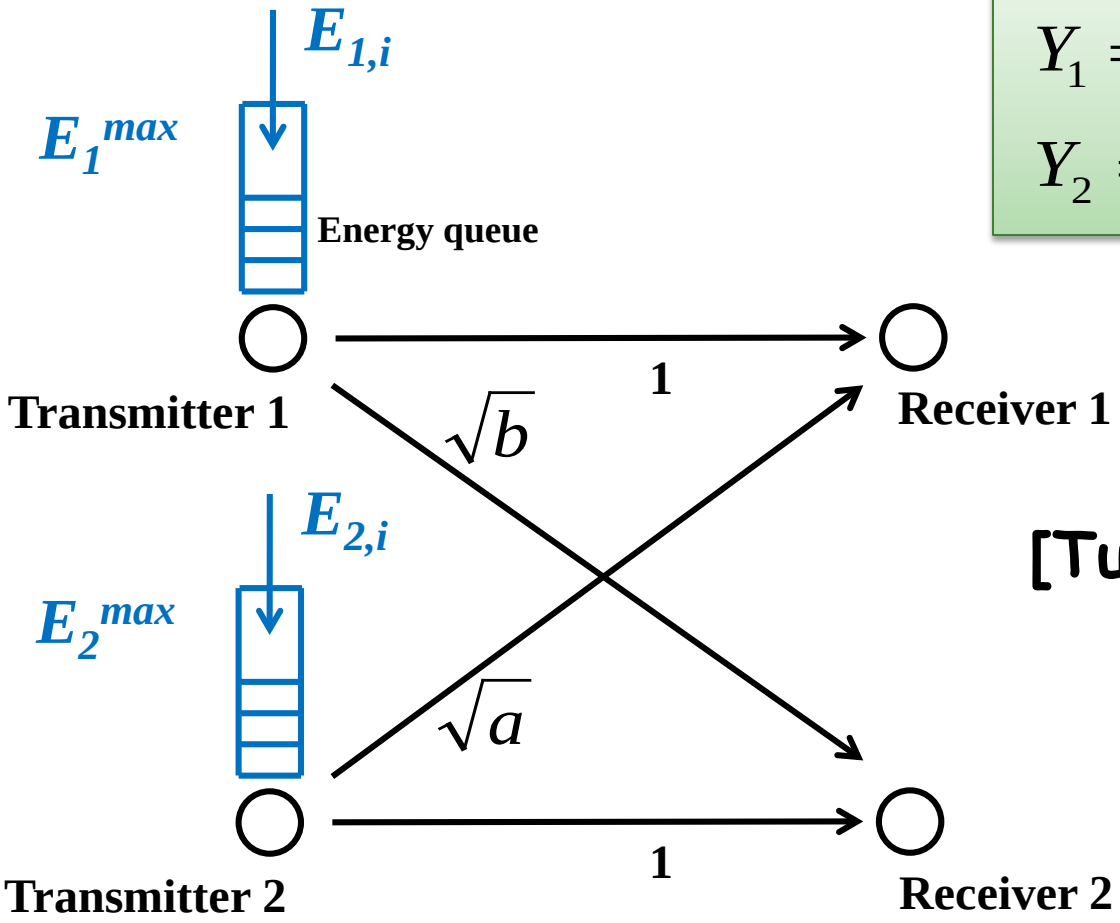


Multiple EH Transmitters

- How to allocate power when there are **more than one energy harvesting transmitters** sharing the same medium?
- How do the **network parameters** affect the optimal policy?
- Many recent multi-node models, e.g., MAC (and BC) [Ozel,Yang,Ulukus'11,'12], Relay [Cui, Zhang,'12], [Oner, Erkip'13], [Varan, Y.'13], …, Two-way Relay [Tutuncuoglu, Varan, Y.'13],…

Interference and EH: Gaussian IC with EH Transmitters

$$Y_1 = X_1 + \sqrt{a}X_2 + Z_1$$
$$Y_2 = \sqrt{b}X_1 + X_2 + Z_2$$



[Tutuncuoglu-Y. '12]

Problem Definition

- **Sum-Throughput Maximization Problem:**

Find optimal transmission power/rate policies that maximize the total amount of data transmitted to both receivers by a deadline of N unit-length time slots.

$$\max_{\mathbf{p_1}\geq 0,\mathbf{p_2}\geq 0} \sum_{i=1}^{N} r(p_{1,i}, p_{2,i})$$

$$s.t. \quad 0 \leq \sum_{i=1}^{n} E_{j,i} - p_{j,i} \leq E_j^{\max}$$

$$j = 1,2 \quad n = 1,\ldots,N$$

Optimal Policy

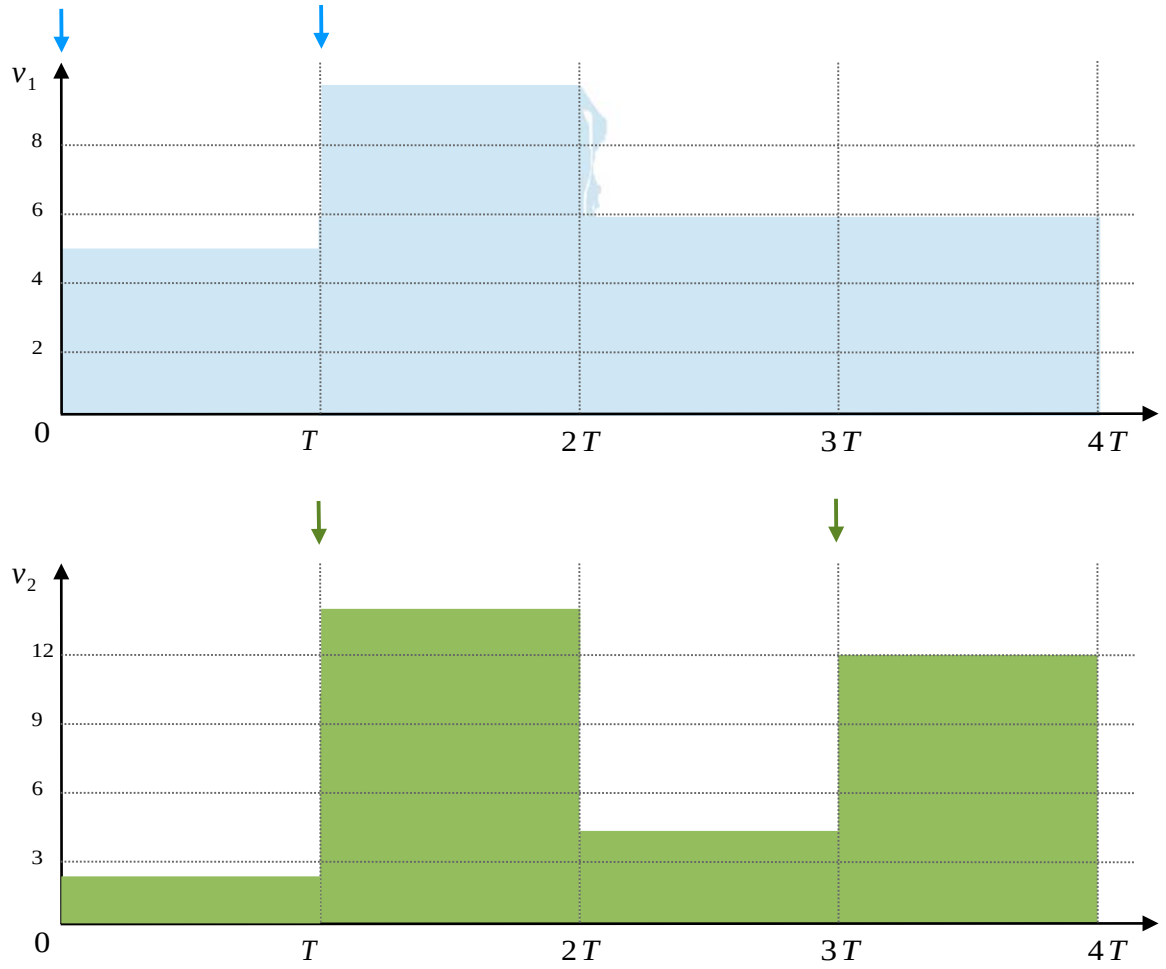
- **Convex problem** allows the solution to be found using **coordinate descent** between $p_{1,i}$ and $p_{2,i}$

Iterative Generalized Directional Water-filling(IGDWF):

constrained water-filling with generalized water levels:

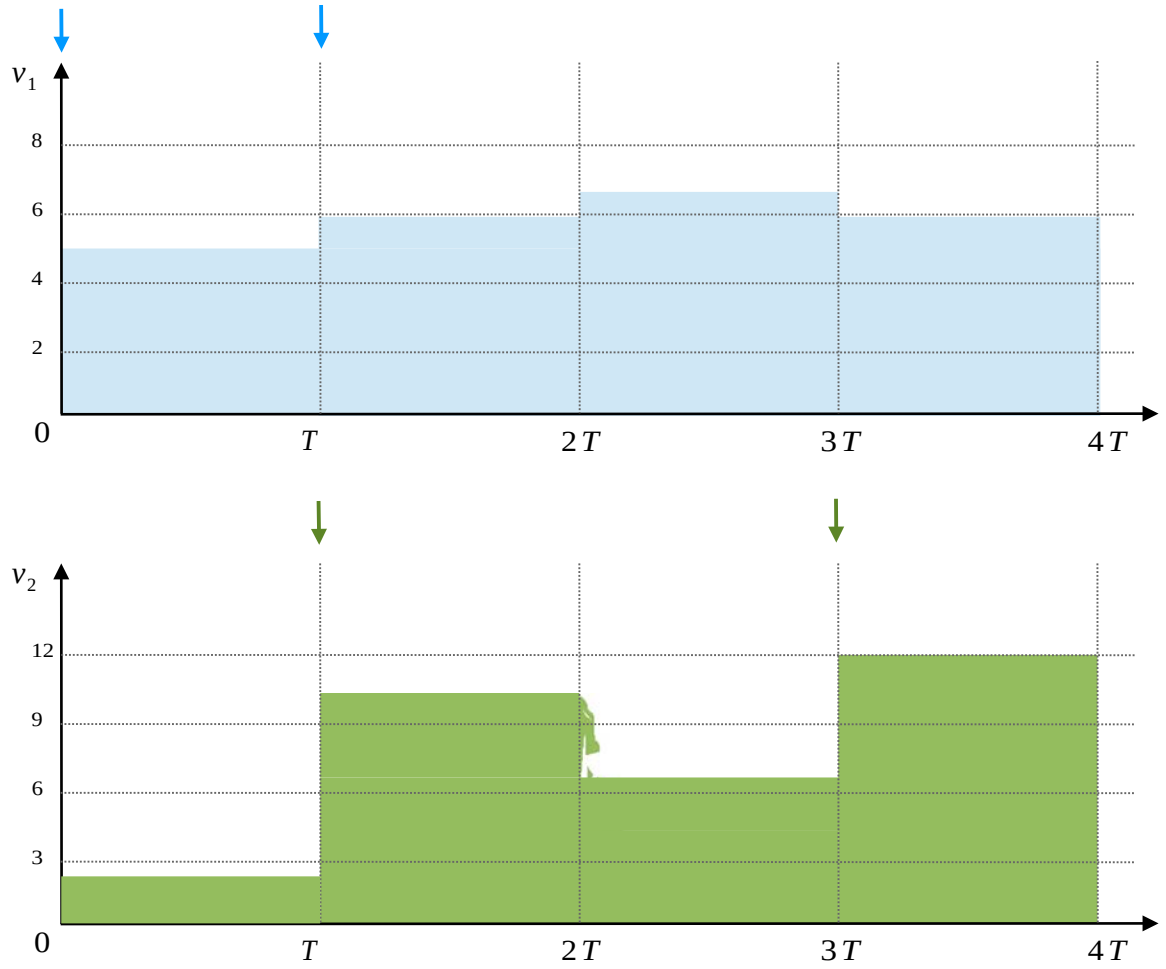
$$v_n = \frac{\partial}{\partial p} r(p)\Big|_{p_n} = \sum_{i=n}^{N} (\lambda_i - \mu_i) - \eta_n$$

Two-user IGDWF



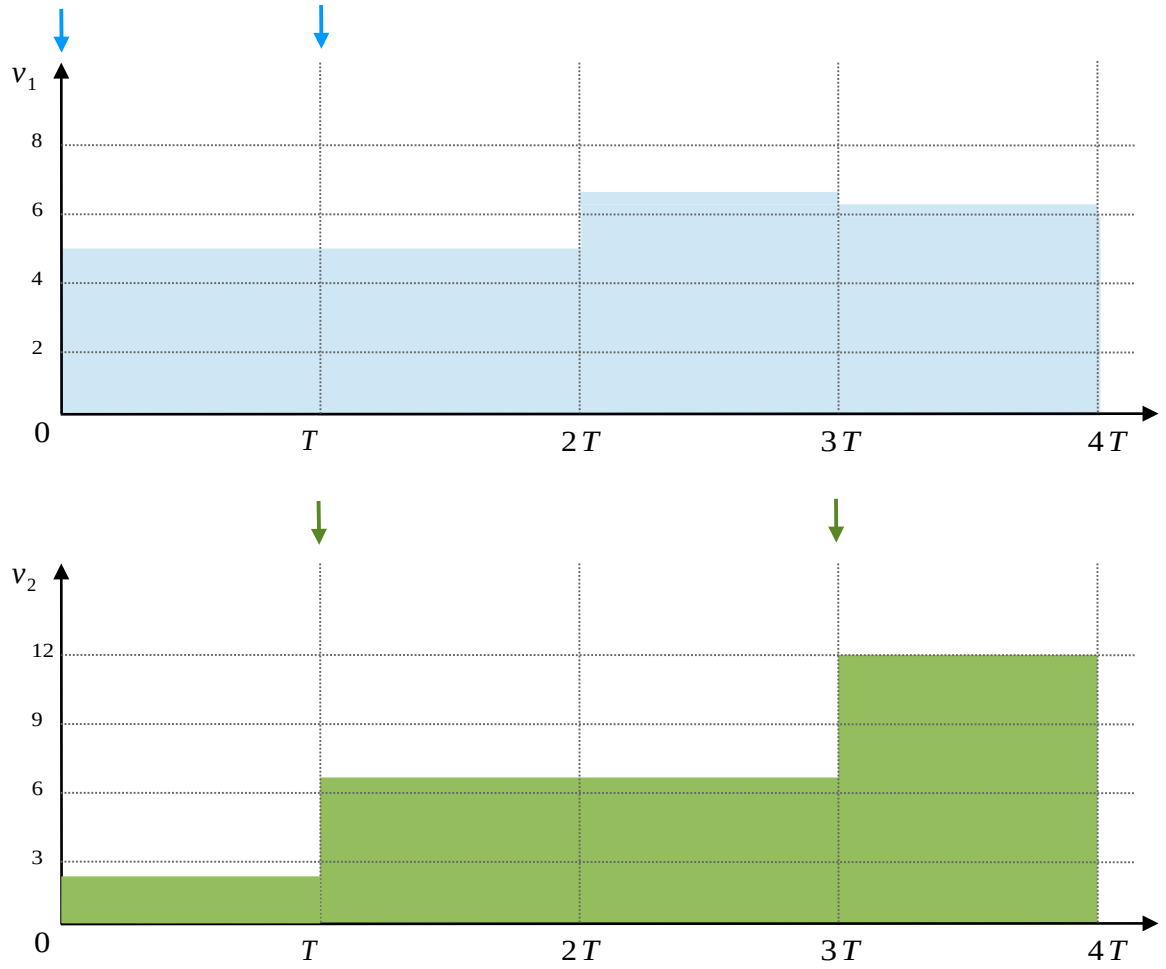
GDWF for User 1

Two-user IGDWF



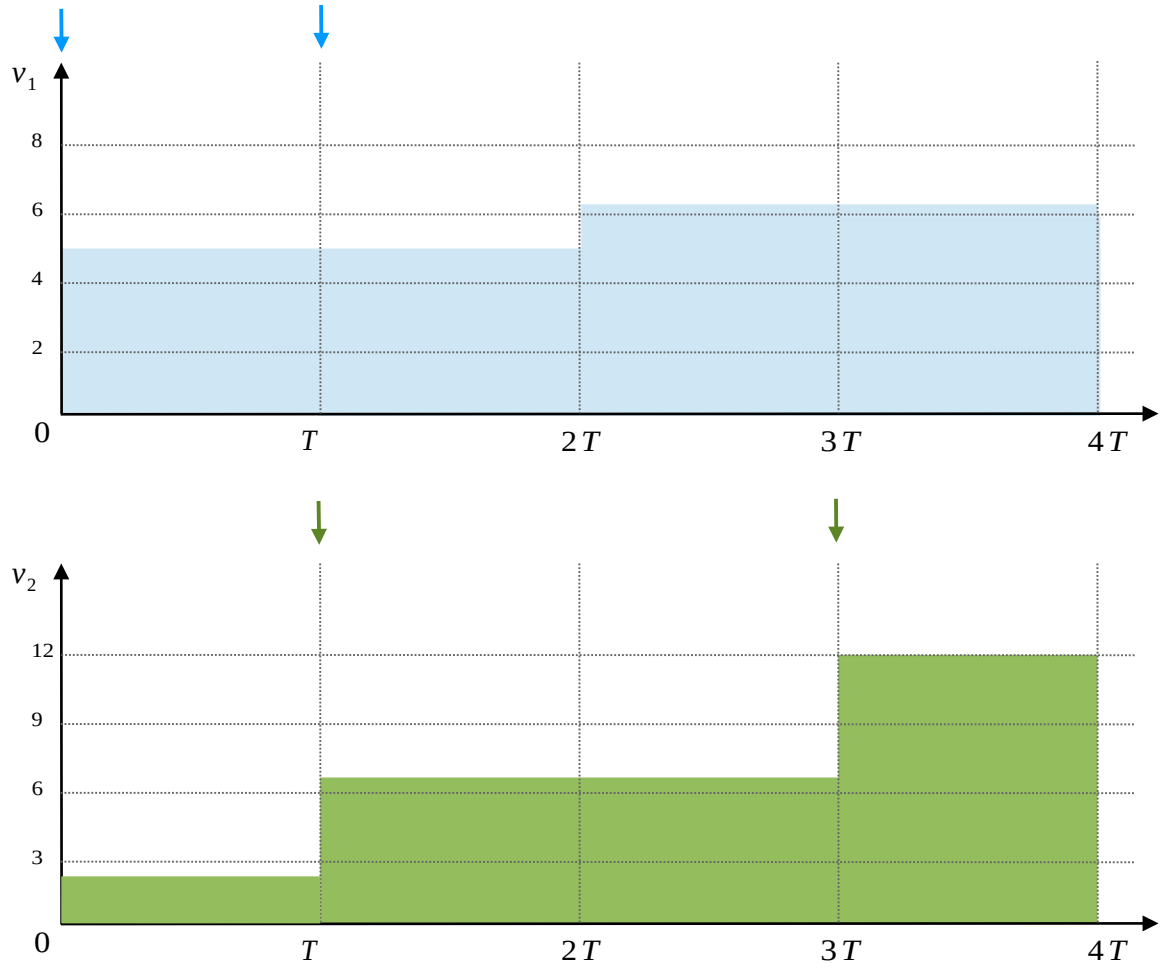
GDWF for User 2

Two-user IGDWF



GDWF for User 1

Two-user IGDWF



GDWF for User 1

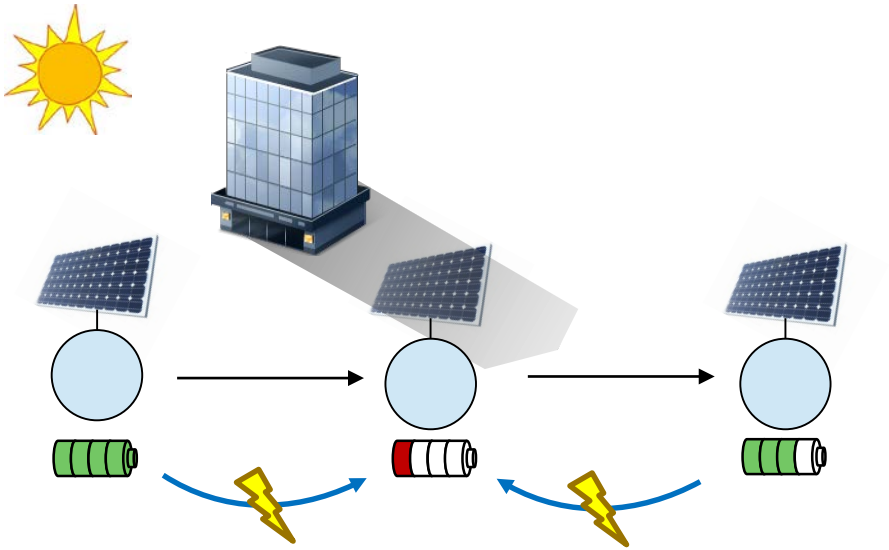
GDWF for User 2

Take-away

- Multiple **energy harvesting transmitters** sharing the same medium: transmit policy of one depends on the others.
- Care need to be exercised in iteratively finding the water-filling solutions.
- Policies do depend heavily on the channels, some instances converge in one iteration and/or result in simplified algorithms, e.g., strong interference.

Multiple EH Transmitters: Energy Cooperation

- **Intermittent energy** $\Rightarrow$ nodes may be **energy deprived!**

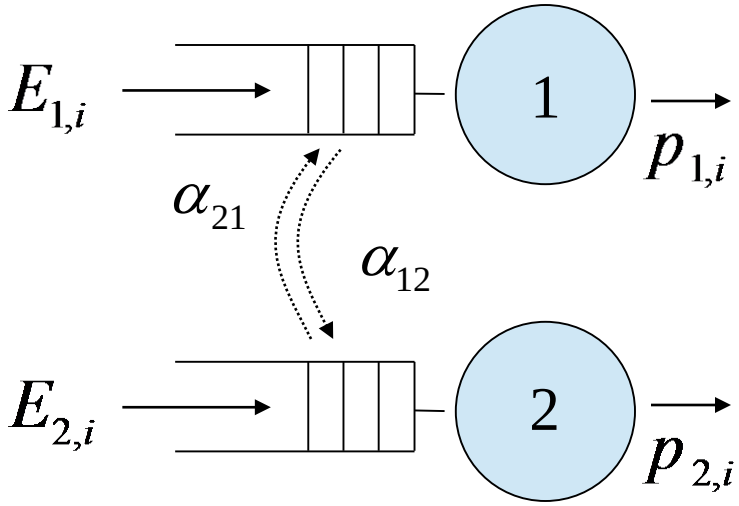


- Energy cooperation between nodes can be very useful!

- [Gurakan-Ozel-Ulukus '12]
- [Tutuncuoglu-Y. '13]

EC-EH

- In time slot i ,transmitter$k$ sends transmitter j an energy of $\delta_{k,j,i}$ with efficiency $\alpha_{k,j}$



- Uni-directional energy transfer is a special case with

$$\alpha_{k,j} = 0, \quad \alpha_{j,k} > 0, \qquad j,k = 1,\ldots,K$$

- Energy in node k's battery at the end of the i^{th} time slot:

$$E_{k,n}^{bat} = \sum_{i=1}^{n}\left(E_{k,i} + \sum_{j=1}^{K}\left(\alpha_{j,k}\delta_{j,k,i} - \delta_{k,j,i}\right) - p_{k,i} \right)$$

Harvested energy

Received and sent energy

Energy used for transmission

EC-EH

Energy Constraints:

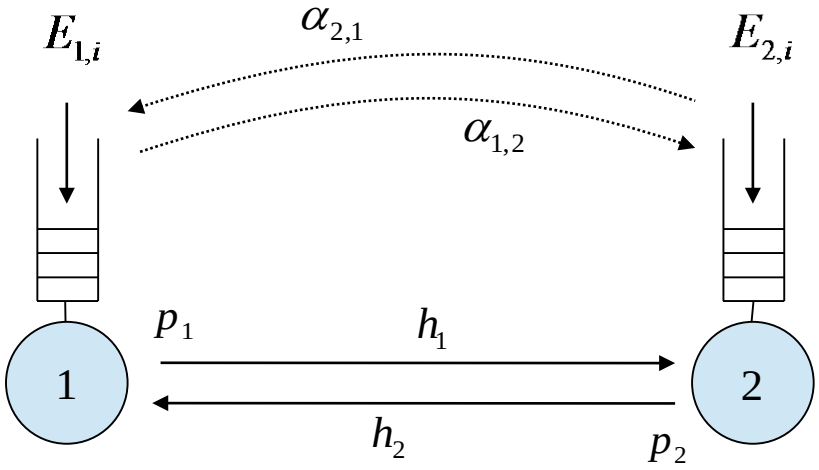
- **Non-negativity** of transmit power and transferred energy: $p_{k,n} \geq 0, \quad \delta_{j,k,n} \geq 0, \quad j,k = 1,\ldots,K, \quad n = 1,\ldots,N$

- **Energy Causality**: Energy required by transmission or transfer is available, i.e., harvested:

$$E_{k,n}^{bat} = \sum_{i=1}^{n} \left(E_{k,i} + \sum_{j=1}^{K} \left(\alpha_{j,k} \delta_{j,k,i} - \delta_{k,j,i} \right) - p_{k,i} \right) \geq 0$$

- **What is the sum-capacity of EC-EH-MAC and EC-EH-T(wo)-W(ay)-C?**

EC-EH-TWC



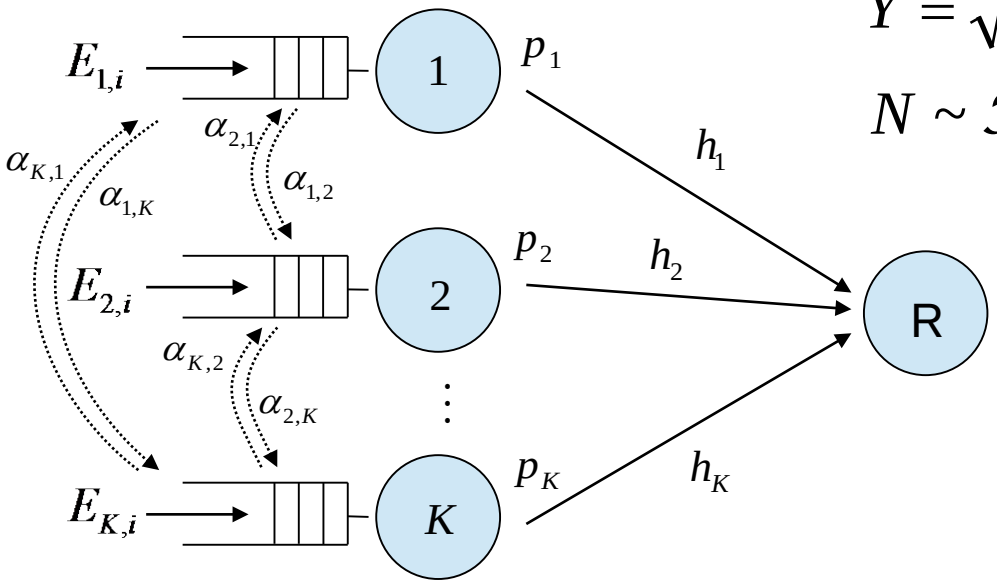
$$Y_1 = X_1 + \sqrt{h_2}\,X_2 + N_1$$

$$Y_2 = X_2 + \sqrt{h_1}\,X_1 + N_2$$

$$N_k \sim \mathcal{N}(0, \sigma_k^2),$$

- **Sum-Capacity:** $C_S^{TWC} = \frac{1}{2}\log(1+p_1) + \frac{1}{2}\log(1+p_2)$

EC-EH-MAC



$$Y = \sqrt{h_1} X_1 + \ldots + \sqrt{h_K} X_K + N$$

$$N \sim \mathcal{N}(0, \sigma^2),$$

Normalize σ^2 and h_k by:

$$Y' = \frac{Y}{\sigma}, \quad X' = X \frac{\sqrt{h_k}}{\sigma}$$

- **Sum-Capacity:** $C_S^{MAC} = \frac{1}{2} \log\left(1 + \sum_{k=1}^{K} p_k\right)$

Problem Statement

[Tutuncuoglu-Y. '13]

- Find maximum achievable sum-rate by optimizing the energy transfer and energy expended for tx.

$$\max_{p_{k,n},\ \delta_{k,j,n}} \sum_{i=1}^{N} C_S(p_{1,i}, p_{2,i}, \ldots, p_{K,i})$$

$$s.t. \quad \delta_{k,j,n} \geq 0, \quad p_{k,n} \geq 0,$$

$$\sum_{i=1}^{n} \left(E_{k,i} + \sum_{j=1}^{K} \left(\alpha_{j,k} \delta_{j,k,i} - \delta_{k,j,i} \right) - p_{k,i} \right) \geq 0$$

$$j,k = 1,\ldots,K, \qquad n = 1,\ldots,N$$

Procrastinating Policies

- **Definition:** *A* **procrastinating policy** satisfies

$$p_{j,i} \geq \sum_{k=1}^{K} \alpha_{k,j} \delta_{k,j,i}$$

i.e., the energy received by a node is not greater than the energy required for transmission within that time slot.

- In a procrastinating policy, a node ***does not transfer energy unless the receiving node intends to use it immediately***.
- **Lemma:** There exists at least one procrastinating policy that solves the sum-capacity problem.
- ➔ Restrict search space to such policies

Sum-Capacity Problem

- Define **consumed powers** $\bar{p}_{k,i} = p_{k,i} + \sum_{j=1}^{K} \delta_{k,j,i} - \alpha_{j,k}\delta_{j,k,i}$
- Sum-Capacity problem can be **decomposed** as

$$\max_{\bar{p}_{k,n}} \sum_{i=1}^{N} C_S^*(\bar{p}_{1,i},\ldots,\bar{p}_{K,i})$$

$$s.t. \quad \bar{p}_{k,n} \geq 0,$$

$$\sum_{i=1}^{n}\left(E_{k,i} - \bar{p}_{k,i}\right) \geq 0$$

$$k = 1,\ldots,K, \quad n = 1,\ldots,N$$

$$C_S^* = \max_{\delta_{k,j,i}} C_S\left(\bar{p}_{k,i} - \sum_{j=1}^{K}(\delta_{k,j,i} - \alpha_{j,k}\delta_{j,k,i})\right)$$

$$s.t. \quad \delta_{k,j,i} \geq 0, \quad \bar{p}_{k,i} \geq \sum_{j=1}^{K}\delta_{k.j,i}$$

$$j,k = 1,\ldots,K$$

Power Allocation
Solved via IGDWF

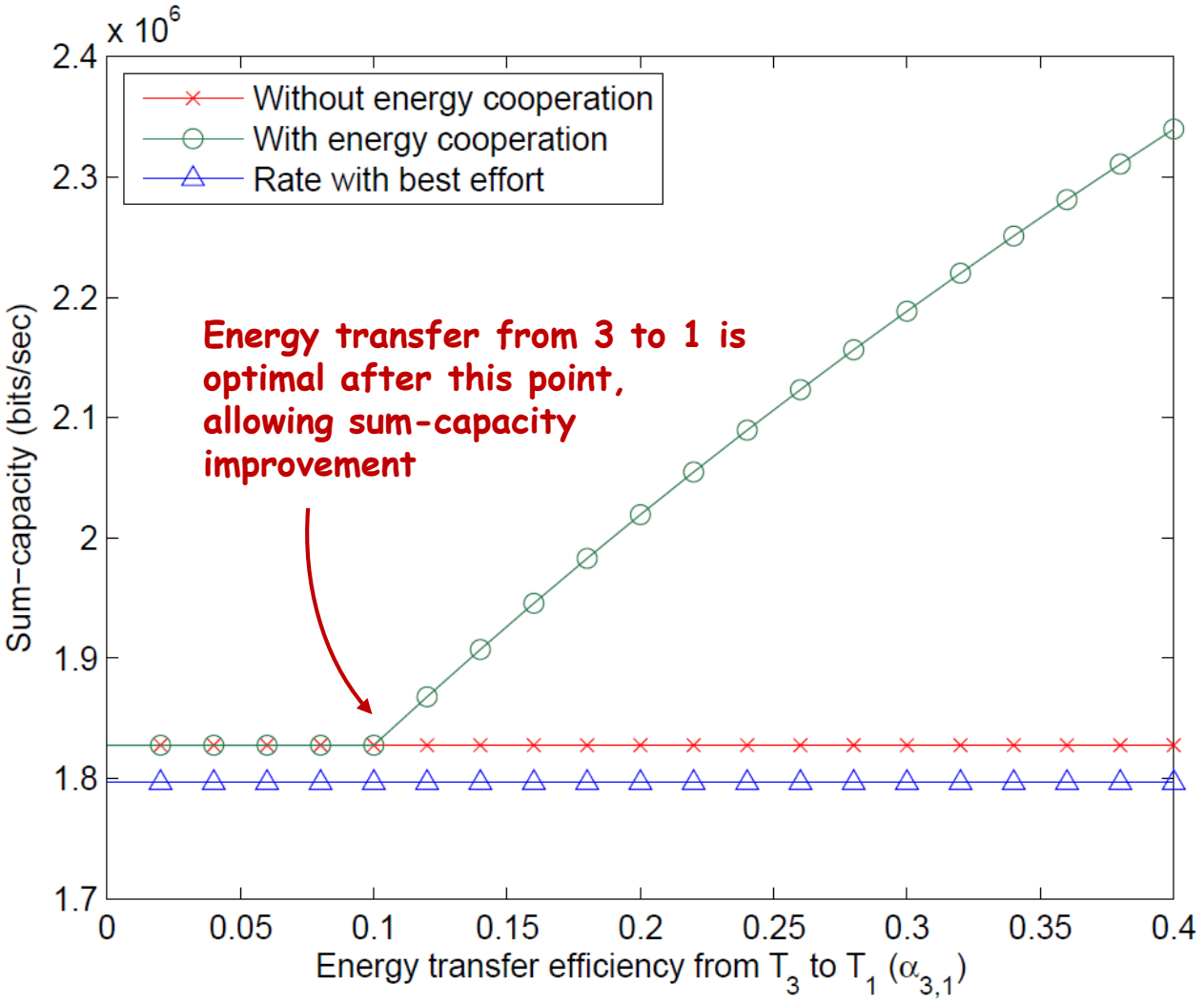
Energy Transfer
Solved directly (single slot)

Simulations

- **MAC**

$$N = 100, \quad T = 1\text{sec},$$
$$h_1 = -100dB,$$
$$h_2 = -105dB,$$
$$h_3 = -110dB,$$
$$N_0 = 10^{-19}\, W / Hz$$
$$E_{1,i} \sim U[0,1]mJ,$$
$$E_{2,i}, E_{3,i} \sim U[0,10]mJ,$$

$$[\alpha_{k,j}] = \begin{bmatrix} 1 & 0.3 & 0.2 \\ 0.1 & 1 & 0.4 \\ \alpha_{3,1} & 0 & 1 \end{bmatrix}$$



Conclusion

- New networking paradigm: **energy harvesting nodes**
- New design insights arise from new energy constraints.
- Realistic concerns, e.g. storage capacity, storage efficiency impact transmission policies.
- Multi-terminal scenarios need to be handled with care.
- **Energy cooperation** brings additional insights and possibilities.
- Information theoretic formulations are challenging but promising to yield new insights.

Current Studies and Open Problems

- Information theoretic limits, optimal coding schemes for energy harvesters
- Operational principles of energy harvesting receivers
- Impact of EH on signal processing PHY algorithms
- Impact on network protocols
- Efficient online schedules, simple practical implementations
- Papers: http://wcan.ee.psu.edu/